

Gauge Theories: Geometry and Cohomological Invariants

M. Kachkachi,¹ A. Lamine,¹ and M. Sarih¹

Received September 25, 1997

We develop a geometrical structure of the manifolds Γ and $\hat{\Gamma}$ associated, respectively, with gauge symmetry and BRST symmetry. Then, we show that $(\hat{\Gamma}, \hat{\zeta}, \Gamma)$, where $\hat{\zeta}$ is the group of BRST transformations, is endowed with the structure of a principal fiber bundle over the base manifold Γ . Furthermore, in this geometrical setup, due to the nilpotency of the BRST operator, we prove that the effective action of a gauge theory is a BRST-exact term up to the classical action. Then, we conclude that the effective action where only the gauge symmetry is fixed is cohomologically equivalent to the action where the gauge and the BRST symmetries are fixed.

1. INTRODUCTION

BRST symmetry was discovered independently by Becchi *et al.* (1974, 1978) and Tyutin (1975) as an invariance of the effective Yang–Mills action. This symmetry is the clue to the proof of renormalizability of a gauge theory and the starting point for the algebraic determination of chiral anomalies (deWitt and Stora, 1984). It was also realized that BRST invariance in quantum field theories is a fundamental requirement for a consistent definition of theories with local gauge invariance (Baulieu, 1985). Indeed, in an invariant gauge theory the gauge degrees of freedom are not physical and must be eliminated. This is done by the usual gauge-fixing procedure in the perturbative Lagrangian approach. One adds to the gauge-invariant Lagrangian a gauge-breaking term rendering the gauge field free propagator well defined and leading to the decoupling of gauge degrees of freedom from the physical ones. This decoupling is guaranteed by the requirement of BRST invariance of the effective action. Hence, gauge symmetry (and its subsequent BRST

¹Département de Mathématiques, F.S.T.S., B.P. 577, Université Hassan 1^{er}, Settat, Morocco.

gauge fixing) gives us an unexpected freedom in defining equivalent formulations of the same physical theory.

In particular, in their original formulation, topological field theories were constructed to have the global symmetry that arises as the BRST symmetry of an appropriate quantum field theory (Witten, 1982). This formulation allows for the possibility of different gauge choices, and these theories are then seen as specific gauge fixings of a higher theory (Schwarz, 1978). So, at the quantum level gauge invariance is broken and it is the BRST invariance which takes place.

By using the nilpotency of the BRST operator we have shown that BRST symmetry cannot be fixed in any way, and any “BRST-fixing condition” is cohomologically equivalent to the gauge-fixing one (Kachkachi and Kachkachi, 1994).

In this paper we develop a geometrical structure of the manifolds Γ and $\hat{\Gamma}$. Then we show that the manifold Γ is a principal fiber bundle over the base space-time manifold M whose structural group is the gauge transformations group. Moreover, we establish that $(\hat{\Gamma}, \hat{\zeta}, \Gamma)$, where $\hat{\zeta}$ is the group of BRST transformations, is a principal fiber bundle over Γ . In this geometrical formalism we associate gauge-fixing conditions $F_1(A) = 0$ and $F_2(A) = 0$, respectively, to surfaces Σ_{F_1} and Σ_{F_2} (over the manifold Γ), which are connected by a gauge transformation, and we associate BRST-fixing conditions $G_1(A, \Psi) = 0$ and $G_2(A, \Psi) = 0$, respectively, to surfaces Σ_{G_1} and Σ_{G_2} (over the manifold $\hat{\Gamma}$), which are connected by a BRST transformation. Furthermore, we establish a cohomological equivalence of a gauge-fixing condition and a BRST-fixing one. Then we get the cohomological equivalence of the effective action and an “effective action whose BRST symmetry is fixed.” This is interpreted by the fact that the BRST symmetry does not impose any more conditions on the theory, and equivalently is always possible (by an appropriate BRST transformation) to go back to the initial gauge condition. These results allow us to conclude that the effective action is the sum of the classical action and a BRST-exact term.

The outline of this paper is as follows. In Section 2 we give the fiber bundle structures of the manifolds Γ and $\hat{\Gamma}$ by introducing a principal fiber bundle (P, G, M) . We show also that a gauge field associated to a connection over P is a local section of the fiber bundle (Γ, ζ, M) . By imposing a gauge-fixing condition $F(A) = 0$, which is represented by a constant function F on the fibers of the fiber bundle Γ , the gauge symmetry is fixed. Then we have to deal with the effective action that encodes the BRST invariance and to consider the geometrical structure of the manifold $\hat{\Gamma}$. This enables us to define the principal fiber bundle $(\hat{\Gamma}, \hat{\zeta}, \Gamma)$ and to identify one of its local sections with the BRST operator Q . In Section 3, by using this geometrical interpretation of the BRST operator, we get the equivalence of two BRST-

fixing conditions defined on the manifold $\hat{\Gamma}$ and we prove that a constraint on Γ is cohomologically equivalent to a constraint on $\hat{\Gamma}$, that is, a $\hat{\Gamma}$ constraint is equal to a Γ constraint plus a Q -commutator term. This cohomological equivalence enables us to prove that the BRST-fixing action is equivalent to the effective action, and hence, the fixation of BRST invariance only adds a Q -commutator term to the effective action. It only changes the gauge-fixing term (does not affect the physical degrees of freedom) and then the BRST symmetry is always present in the effective action. In Section 4 we illustrate our results by the case of Yang–Mills theories and we stress that our formalism is general for any gauge theories. Section 5 is devoted to our conclusion and open problems.

2. FIBRATION OF THE MANIFOLDS Γ AND $\hat{\Gamma}$

2.1. Gauge Fields and Gauge Transformations

We consider a G -principal fiber bundle (P, G, M) over the manifold M , with G a compact Lie group, endowed with a local trivialization (U_i, φ_i) such that, for each i , the application $\tau_i: M \rightarrow P$ given by $\tau_i(p) = \varphi_i^{-1}(p, e)$, where e is the unit element of G , defines a local section of the fiber bundle (P, G, M) . Next, we introduce a connection ω on the fiber bundle P as follows:

$$A_i = \tau_i^* \omega \quad (2.1)$$

where (A_i) are Lie G -valued one-forms over the open sets (U_i) and τ_i^* is the pullback of τ_i . The one-forms (A_i) satisfy the following relations (Schwarz, 1978):

$$A_j = \Psi_{ij}^{-1} A_i \Psi_{ij} + \Psi_{ij}^{-1} d\Psi_{ij} \quad (2.2)$$

with $\Psi_{ij}: U_i \cap U_j \rightarrow G$ are transition functions of the fiber bundle (P, G, M) . Reciprocally we have the following result (Nakahara, 1990):

Proposition 1. If the one-forms A_i , which are Lie G -valued and defined on the open set U_i , satisfy equation (2.2), then there exists one and only one connection ω satisfying the relation $A_i = \tau_i^* \omega$ over U_i .

Consequently we have the following:

Corollary 1. Let us consider two local sections τ_1 and τ_2 such that

$$\tau_2(p) = \tau_1(p)g(p) \quad (2.3)$$

where $g: M \rightarrow G$. Then the associated local forms A_1 and A_2 satisfy

$$A_2 = g^{-1}A_1g + g^{-1}dg \quad (2.4)$$

However, in the geometrical interpretation in terms of the fiber bundle structure, gauge fields denoted A_i are viewed as one-forms over the base manifold M and satisfy a relation analogous to equation (2.4). Moreover, the gauge transformation group ζ is the group of vertical automorphisms φ over the fiber bundle (P, G, M) that satisfy the base condition

$$\varphi(p) = p \quad (2.5)$$

for some p . In other words, a gauge transformation is an automorphism which commutes with the action of G ,

$$\varphi(gp) = g\varphi(p) \quad (2.6)$$

Locally, gauge transformations are specified by the following proposition (Nakahara, 1990):

Proposition 2. A gauge transformation f is completely determined by a family of applications (α_i) such that

$$\alpha_i = \varphi_i f \tau_i \quad (2.7)$$

where $\varphi_i \in C^\infty(U_i, G)$ and $\alpha_j = ad(\Psi_{ij}^{-1})\alpha_i$. Here ad is the adjoint representation of the group G and (Ψ_{ij}) are transition functions of (P, G, M) . Reciprocally, a family (α_i) of applications satisfying equation (2.7) defines a gauge transformation

$$f = (\tau_i \pi)[(\alpha_i \pi) \varphi_i] \quad (2.8)$$

over $\pi^{-1}(U_i)$. Moreover, by expressing explicitly $f^*\omega$, we get the gauge transformation action f on a connection ω and then, by using the relation (2.1), we express its action on the gauge field A as follows (Nakahara, 1990; Birmingham *et al.*, 1991)

$$A' = f^{-1}Af + f^{-1}df \quad (2.9)$$

Then, the gauge transformation group is realized on the the manifold M as diffeomorphism group.

2.2. The Fiber Bundle (Γ, ζ, M)

The manifold Γ can be equipped with a principal fiber bundle structure over the manifold M whose orbits are gauge orbits and whose structural group is the gauge transformations group. Indeed, we define the canonical projection of Γ over M by

$$\begin{aligned} \tilde{\pi}: \Gamma &\rightarrow M \\ A(x) &\rightarrow \tilde{\pi}(A(x)) = x \end{aligned} \quad (2.10)$$

and we consider the gauge field as a map: $M \rightarrow \Gamma$ and as a local section of the fiber bundle (Γ, ζ, M) . Equation (2.9) shows also that the action of ζ on Γ is free (does not have any fixed point). Moreover, Γ is locally trivial: let (U_i, φ_i) be a local trivialization of the fiber bundle P . Then, the family $(\tilde{\pi}^{-1}(U_i), \tilde{\varphi}_i)$ is a local trivialization of Γ such that

$$\begin{aligned} \tilde{\varphi}_i: \tilde{\pi}^{-1}(U_i) &\rightarrow U_i \times \zeta \\ A &\rightarrow (\tilde{\pi}(A), \tilde{\varphi}_i(A)) \end{aligned} \tag{2.11}$$

where $\tilde{\varphi}_i(A)$ is the matricial representation of the gauge field in $\text{Lie}G$, i.e.,

$$A = A_\mu(x) dx^\mu = A_\mu^a(x) T^a dx^\mu \tag{2.12}$$

and T^a is a $\text{Lie}G$ basis. Moreover, we define the transition functions of the fiber bundle (Γ, ζ, M) as the applications

$$\begin{aligned} \Psi_{ij}: U_i \cap U_j &\rightarrow \zeta \\ x &\rightarrow Ad(\Psi_{ij}^{-1}(x)) \end{aligned} \tag{2.13}$$

where Ad is the adjoint representation of $\text{Lie}G$. Then we consider a system (A^1, \dots, A^n) of gauge fields on the manifold Γ (which represents the gauge orbits) such that

$$\forall A \in \Gamma \quad \exists (m_i) / A = m_i A^i \tag{2.14}$$

and hence, (x_μ, m_i) defines a system of local coordinates on Γ .

2.3. BRST Transformations

The BRST quantization of gauge theories is stated by introducing a gauge-fixing term in the Lagrangian to get the free propagators of gauge fields well defined. This fixing term implies the presence of nonphysical degrees of freedom in the theory, but they are canceled by ghosts fields. Then the effective action is given by

$$S_{\text{eff}} = S_0 + S_{\text{FP}} + S_{\text{GF}} \tag{2.15}$$

where S_0 , S_{GF} , and S_{FP} are, respectively, the classical action, the gauge-fixing action, and the ghost action. However, even if the gauge invariance is fixed at this level, another symmetry of the effective action appears, that is, the BRST symmetry generated by the nilpotent operator Q . This operator can be decomposed as follows:

$$Q = d + \delta \tag{2.16}$$

such that $Q^2 = 0$ and $d^2 = \delta^2 = \delta d + d\delta = 0$. Here d is the exterior derivative over the manifold M (or section of Γ) and δ is the restriction of

Q to the fibers of Γ . Furthermore, the cohomological groups associated to the operators d and δ are given by (Kachkachi and Kachkachi, 1994)

$$\begin{aligned} H^0(d) &= C^\infty\left(\frac{\Gamma}{P_0}\right) \\ H_0(\delta) &= \left(\frac{\ker\delta}{\text{Im}\delta}\right) = C^\infty(\Gamma) \end{aligned} \tag{2.17}$$

where P_0 is a gauge orbit. The BRST transformations of collections (A, Ψ) are defined as (Birmingham *et al.*, 1991)

$$\begin{aligned} [Q, A] &= \Psi \\ [Q^2, A] &= \{Q, [Q, A]\} = -D\phi \end{aligned} \tag{2.18}$$

where $D\phi$ is the covariant derivative of a gauge parameter ϕ . Locally, a BRST transformation is considered as a gauge transformation where the gauge parameter is replaced by an anticommuting one. Also, we can see from the splitting (2.16) of the BRST operator Q that the group of gauge transformations is a subgroup of the BRST transformation group. We will see in Section 2.5 that the group of BRST transformations $\hat{\zeta}$ is the structural group of the fiber bundle $(\hat{\Gamma}, \hat{\zeta}, \Gamma)$.

2.4. Geometrical Interpretation of Ghost Fields

Let us consider a chart r defined by

$$\begin{aligned} r: U \times V &\rightarrow P \\ (x, y) &\rightarrow r(x, y) = \phi_i^{-1}(a(x), g(y)) \end{aligned} \tag{2.19}$$

where $a: M \rightarrow M$ and $g: G \rightarrow G$, and let us consider a connection ω on the fiber bundle (P, G, M) which is expressed on the chart r as follows:

$$\omega(x, y) = ad(g(y)^{-1})A_\mu(x) dx^\mu + g^{-1}(y)\partial_\alpha g(y) dy^\alpha$$

Its vertical part can be rewritten as

$$\begin{aligned} C(y) &= C_\alpha(y) dy^\alpha \\ C_\alpha(y) &= g(y)^{-1}\partial_\alpha g(y) \end{aligned} \tag{2.20}$$

Since $C_\alpha(y) \in \text{Lie}G$, it can be written as $C_\alpha(y) = C_\alpha^a(y)T^a$. At this level, we identify the usual FP fields with the real one-forms $C^a(y) = C_\alpha^a dy^\alpha$ and the ghost field Ψ with its covariant derivative,

$$\Psi_\mu = -D_\mu C$$

Hence, due to the fact that the operator Q is the exterior derivative on the manifold Γ , the ghost field appears as a Q -exact term on the fiber bundle (Γ, ζ, M) , i.e., $\Psi = [Q, A]$, which expresses the BRST transformation of the field A . Here we note that, because

$$\begin{aligned} H^0(Q) &= \{\text{equivalence classes of physical observables}\} \\ &= C^\infty\left(\frac{\Gamma}{P_0}\right) = \frac{ImQ}{kerQ} \end{aligned}$$

the introduction of ghost fields in the theory does not affect the physical observables.

2.5. Fiber Bundle Structure of the Manifold $\hat{\Gamma}$

As we have noted before, the BRST operator Q is the exterior derivative on the manifold Γ and can be written locally in the form

$$Q = dm_i \frac{\partial}{\partial m_i}, \quad [Q, m_i] = dm_i \tag{2.21}$$

The manifold $\hat{\Gamma}$ is defined as the set of all collections (A, Ψ) (over which the effective action is BRST invariant) and then can be endowed with a fiber bundle structure.

Proposition 3. The manifold $\hat{\Gamma}$ is a principal fiber bundle over the manifold Γ whose structural group is the BRST transformation group. Then, the canonical projection of $\hat{\Gamma}$ over Γ is defined by

$$\begin{aligned} \hat{\Pi}: \hat{\Gamma} &\rightarrow \Gamma \\ (A, \Psi) &\rightarrow \hat{\Pi}(A, \Psi) = A \end{aligned} \tag{2.22}$$

Also, Q and Ψ are local sections of the fiber bundle $(\hat{\Gamma}, \hat{\zeta}, \Gamma)$. Indeed, the local trivialization of $\hat{\Gamma}$ is defined by the family $(\hat{\Pi}^{-1}(\Pi^{-1}(U_i)), \hat{\phi}_i)$ such that

$$\begin{aligned} \hat{\phi}_i: \hat{\Pi}^{-1}(\Pi^{-1}(U_i)) &\rightarrow \Pi^{-1}(U_i) \times \hat{\zeta} \\ (A, \Psi) &\rightarrow (A, \hat{\phi}_i(\Psi)) \end{aligned} \tag{2.23}$$

where $\hat{\phi}(\Psi)$ is the matricial representation of the ghost field in the Lie algebra of the group ζ . Then, the transition functions of the fiber bundle $\hat{\Gamma}$, denoted by $\hat{\Psi}_{ij}$, are defined as follows:

$$\begin{aligned} \hat{\Psi}_{ij}: \Pi^{-1}(U_i) \cap \Pi^{-1}(U_j) &\rightarrow \zeta \\ A &\rightarrow ad(\hat{\Psi}_{ij}(A)) \end{aligned} \tag{2.24}$$

where ad is the adjoint representation of the group ζ .

3. COHOMOLOGICAL EQUIVALENCE BETWEEN Γ AND $\hat{\Gamma}$

In geometrical language the quantization of gauge theories, which is done at the first step by breaking the gauge symmetry, is equivalent to choosing only one representative from each gauge equivalent class, i.e., picking out a point from each gauge orbit and considering a constraint surface Σ_F on the manifold Γ defined by the equation

$$F(A) = 0 \quad (3.1)$$

with F a map $\Gamma \rightarrow \text{Lie}G$. This is equivalent to choosing a local section of the fiber bundle (Γ, ζ, M) which is in our setting a gauge-fixing condition. This implies that the surface Σ_F meets every orbit in one and only one point. This assertion induces the following lemma:

Lemma 1. All gauge-fixing conditions are equivalent and then two constraint surfaces are related by a gauge transformation. Indeed, let us consider Σ_{F_1} and Σ_{F_2} . Since F_1 and F_2 are local sections of Γ , then there exists an element of the structural group ζ of the fiber bundle (Γ, ζ, M) connecting the two surfaces, i.e., $\exists f \in \zeta / \Sigma_{F_2} = f^* \Sigma_{F_1}$.

In the same way, we define a BRST-fixing condition on $\hat{\Gamma}$ by the equation

$$G(A, \Psi) = 0 \quad (3.2)$$

where $G: \hat{\Gamma} \rightarrow \text{Lie}\hat{\zeta}$. This is associated to a local section on $\hat{\Gamma}$. Furthermore, we associate to equation (3.2) a G -constraint surface $\hat{\Sigma}_G$ defined on the manifold $\hat{\Gamma}$. Then, using the fibration $(\hat{\Gamma}, \hat{\zeta}, \Gamma)$ we have the following lemma:

Lemma 2. Let us consider a map $G(m_i, \hat{m}_i)$ over $\hat{\Gamma}$. It is constant on the BRST orbits. Then $G(m_i, \hat{m}_i)$ takes the form

$$G(m_i, \hat{m}_i) = G_0(m_i) + G_1(m_i)\hat{m}_i + \dots \quad (3.3)$$

where G_0 and G_1 are maps on Γ . Indeed, since $G(m_i, \hat{m}_i)$ is constant on the BRST fibers, then it projects into a map on Γ , i.e.,

$$\hat{\Pi}(G(m_i, \hat{m}_i)) = G_0(m_i) \quad (3.4)$$

and it decomposes into G_0 and a term depending only on orbits parameters, say \hat{m}_i . So, we have $G(m_i, \hat{m}_i) = G_0(m_i) + G_1(m_i)\hat{m}_i$. Finally, equation (3.3) and the nilpotency property of the operator Q enable us to get the following proposition.

Proposition 4. A BRST-fixing condition on the manifold $\hat{\Gamma}$ is equivalent to a gauge-fixing condition on the manifold Γ up to a Q -exact term. Furthermore, if $G(A, \Psi)$ is a BRST-fixing condition, then it takes the form

$$G(A, \Psi) = G_0(A) + \{Q, \Lambda\} \quad (3.5)$$

where Λ is an arbitrary function of the field Ψ . Indeed, the action of the operator Q on (3.5) is expressed as

$$\{Q, G(A, \Psi)\} = \{Q, G_0(A)\} + \{Q, G_1(A)\Psi\} \quad (3.6)$$

and we have

$$\begin{aligned} \{Q, G_1(A)\Psi\} &= G_1(A)\{Q, \Psi\} + \{Q, G_1(A)\}\Psi \\ &= G_1(A)\{Q, [Q, A]\} + \frac{\partial G_1(A)}{\partial A} \Psi^2 \end{aligned} \quad (3.7)$$

The coefficient of G_1 of the first term (in the last equation) is the Jacobi identity, which is equal to zero. Furthermore, $\Psi^2 = 0$ (because Ψ is an anticommuting parameter) and then we have

$$\{Q, G(A, \Psi)\} = \{Q, G_0(A)\} \quad (3.8)$$

So, we conclude that $G(A, \Psi)$ and $G_0(A)$ are equivalent up to a Q -exact term, that is, $G(A, \Psi) = G_0(A) + \{Q, \Lambda\}$. This means that the F constraint and the G constraint defined, respectively, on Γ and on $\hat{\Gamma}$ are cohomologically equivalent and that the projection of the G -constraint surface on the manifold Γ gives an F -constraint surface:

$$\hat{\Pi}(\hat{\Sigma}_G) = \Sigma_F \quad (3.9)$$

The splitting of the BRST operator $Q = d + \delta$ implies the following proposition.

Proposition 5. The effective action is cohomologically equivalent to the classical action:

$$S_{\text{eff}} = S_0 + \{Q, \Lambda\} \quad (3.10)$$

Indeed the decomposed expression of Q gives

$$\{Q, S_0\} = dS_0 \quad (3.11)$$

Furthermore, the gauge invariance of the classical action implies that

$$\{Q, S_0\} = 0 \quad (3.12)$$

However, the BRST invariance is expressed as

$$\{Q, S_{\text{eff}}\} = 0 \quad (3.13)$$

Consequently, we have the following corollary:

Corollary 2. All the effective actions are equivalent. Indeed, if S'_{eff} is an other effective action, then (3.10) implies that

$$S'_{\text{eff}} = S_0 + \{Q, \Lambda'\} \quad (3.14)$$

and we have

$$\begin{aligned} S'_{\text{eff}} &= S_{\text{eff}} + \{Q, \Lambda' - \Lambda\} \\ &= S_{\text{eff}} + \{Q, \Lambda''\} \end{aligned} \quad (3.15)$$

Finally, this algebraic treatment effectively shows that the BRST symmetry cannot be fixed at the quantum level and only the gauge symmetry can be fixed.

4. YANG-MILLS THEORY

The effective action associated to the classical action of a free Yang-Mills theory, when considering a gauge fixing condition $F(A) = 0$, is given by (Gieres, 1988).

$$S_{\text{eff}} = \int d^4x \left(L_{\text{class}} - \frac{1}{2\lambda} F^a F_a + \varpi^a(QF^a) \right) \quad (4.1)$$

Let $G(A, \Psi) = 0$ be a BRST-fixing condition; then (3.5) gives

$$G(A, \Psi) = F(A) + \{Q, \Lambda\} \quad (4.2)$$

or

$$G(A, \Psi) = G_0(A) + G_1(A)\Psi \quad (4.3)$$

Then replacing the expression of the F constraint given by (4.2) and (4.3) in equation (4.1), we get

$$\begin{aligned} S_{\text{eff}} &= \int d^4x \left[L_{\text{class}} - \frac{1}{2\lambda} (G^a(A, \Psi) - \{Q, \Lambda^a\})(G_a(A, \Psi) - \{Q, \Lambda_a\}) \right. \\ &\quad \left. + \varpi^a(QG_a(A, \Psi) - \{Q, \Lambda_a\}) \right] \\ &= \int d^4x \left[L_{\text{class}} - \frac{1}{2\lambda} G^a G_a + \varpi^a \{Q, G_a\} + \frac{1}{2\lambda} (G^a \{Q, \Lambda\} \right. \\ &\quad \left. + \{Q, \Lambda^a\} G_a) - \varpi^a \{Q, \{Q, \Lambda_a\}\} + \frac{1}{2\lambda} \{Q, \Lambda^a\} \{Q, \Lambda_a\} \right] \end{aligned} \quad (4.4)$$

Furthermore, the nilpotency of the operator Q implies that $\{Q, \{Q, \Lambda\}\} = 0$ and

$$\{Q, \Lambda\} \{Q, \Lambda\} = \Psi \frac{\partial \Lambda}{\partial A} \Psi \frac{\partial \Lambda}{\partial A} = \left(\frac{\partial \Lambda}{\partial A} \right)^2 \Psi^2 = 0$$

since $Q = \Psi \partial/\partial A$. Otherwise,

$$\begin{aligned} G(A, \Psi)\{Q, \Lambda\} + \{Q, \Lambda\}G(A, \Psi) &= (G_0(A) + G_1(A))\Psi \frac{\partial \Lambda}{\partial A} \\ &\quad + \Psi \frac{\partial \Lambda}{\partial A} (G_0(A) + G_1(A))\Psi \\ &= G_0\Psi \frac{\partial \Lambda}{\partial A} + \Psi \frac{\partial \Lambda}{\partial A} G_0 \\ &= G_0\{Q, \Lambda\} \\ &= \{Q, \Lambda'\} \end{aligned} \tag{4.5}$$

Then, S'_{eff} is expressed in terms of S_{eff} as follows:

$$S_{\text{eff}} = S'_{\text{eff}} + \{Q, \Lambda'\} \tag{4.6}$$

where

$$S'_{\text{eff}} = \int d^4x \left[L_{\text{class}} - \frac{1}{2\lambda} G^a G_a + \omega^a \{Q, G_a\} \right] \tag{4.7}$$

5. CONCLUSION AND PERSPECTIVES

In this paper we have established explicitly the fiber bundle structure of the manifolds Γ and $\hat{\Gamma}$ associated, respectively, to the gauge symmetry and to the BRST symmetry. Furthermore, we have shown that the gauge-fixing term (on Γ) and the BRST-fixing term (on $\hat{\Gamma}$) are cohomologically equivalent. Hence the projection of the second condition on Γ gives the first one. This enables us to get the effective action as the sum of the classical action and of a Q -exact term.

One can try to extend this formalism to topological field theories and then consider the metric tensor on the two manifolds.

REFERENCES

- Baulieu, L. (1985). *Physics Reports*, **129C**, 1.
- Becchi, C., Rouet A., and Stora, R. (1974). *Physics Letters*, **52B**, 344.
- Becchi, C., Rouet A., and Stora, R. (1978). *Annals of Physics*, **98**, 287.
- Birmingham, D., Blau, M., Rakowski, M., and Thompson, G. (1991). *Physics Reports*, **209**, 129.
- deWitt, B., and Stora, R., eds. (1984). *Relativity, Groups, and Topology II (Les Houches, Session XL)*, North-Holland, Amsterdam.
- Gieres, F. (1988). *Geometry of Supersymmetric Gauge Theories*, Springer-Verlag, Berlin.
- Kachkachi, H., and Kachkachi, M. (1994). *Journal of Mathematical Physics*, **35**, 4467.
- Nakahara, M. (1990). *Geometry, Topology and Physics*, Hilger, Bristol.
- Schwarz, A. S. (1978). *Letters in Mathematical Physics*, **2**, 247.
- Tyutin, I. V. (1975). *Trudy Fizicheskogo Instituta Akademiia Nauk SSSR*, **39**.
- Witten, E. (1982). *Journal of Differential Geometry*, **17**, 661.
- Witten, E. (1991). In *Trieste Conference on Topological Methods in Quantum Field Theories, Trieste, Italy, 1990*, W. Nahm, S. Randjbar-Daemi, E. Sezgin, and E. Witten, eds., World Scientific, Singapore.